Solution –Key
STRENGTH OF MATERIALS
B.TECH. IV SEMESTER, CIVIL ENGINEERING

SECTION – A

Q.1: Fill up the blanks with suitable answer:

i) Unwin's formula for the diameter 'd' in mm of rivet for a given plate thickness 't' in mm is expressed as **$d=6.04\sqrt{t}$**

ii) Maximum permissible value of working stress in welds for bending stress in tension or compression is **0.66 fy**

iii) If A & B are Lame's constants for a thick cylinder then the hoop stress at radius 'r' is **$(B / r^2) + A$**

iv) Slenderness of a column of effective length l and least radius of gyration is **l / k**

v) In a cantilever of length l carrying a load whose intensity varies uniformly from zero at the free end to w per unit run at the fixed end, the maximum B.M. is **$(wl^2)/6$**

vi) Ratio of the Shear stress at Neutral axis and maximum Shear stress for a triangular cross section with base parallel to horizontal is **8/9**

vii) Maximum deflection of a cantilever of span 'L' carrying a point load 'w' at a distance 'a' from the fixed end is **$(wa^2)(3L- a)/(6EI)$**

viii) The ratio of Maximum Shear Stress in the rectangular section and in the circular section is **1.50:1.33**

ix) In a simply supported beam carrying a load whose intensity varies uniformly from zero at one end to w per unit run at the mid span, the maximum B.M. is **$(wl^2)/12$**

x) If 'n' is the number of revolution per minute and 'T' is in Nm, then the Power transmitted 'P' by a shaft is equal to **$[\pi nT / 30]W$**

SECTION – B

Q.2) a) At a point in a piece of elastic material there are three mutually perpendicular planes on which the stresses are as follows: Tensile stress 50N/mm^2 and shear 40N/mm^2 on one plane, compressive stress 35 N/mm^2 and complementary shear stress 40 N/mm^2 on the second plane. No stress on the third plane. Find i) the positions of the planes on which there is no stress ii) the principal stresses and the positions of the planes on which they act. Use Mohr's circle of stress construction method or otherwise.

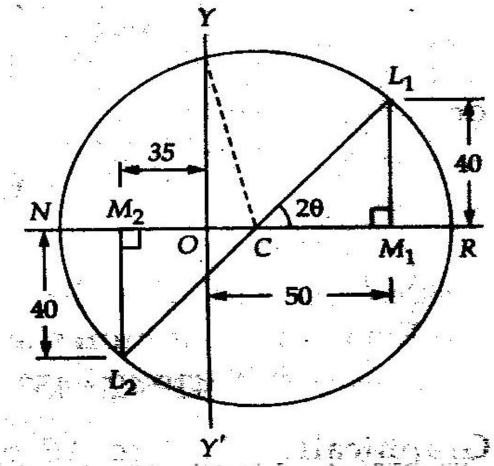
Solution:

(i) Mathematically

Given $\sigma_x = +50\ \text{N/mm}^2$

$\sigma_y = -35\ \text{N/mm}^2$

$\tau_{xy} = 40\ \text{N/mm}^2$



(a) For principal plane using the relation:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

We have, $\tan 2\theta = \dfrac{2 \times 40}{50 + 35} = 0.9412$ ...(Compressive σ_y is taken as –ve)

or $2\theta = 43.264°$

or $\theta = 21.632° = 21°\ 38'$

So, the main principal plane makes an angle of 21°38′ with the 50 N/mm^2 stress. The other two principal planes are mutually at right angles with the main plane, *i.e.*, at 90°+21°38′ (= 111°38′) and 180° + 21°38′(= 201°38′)

(*b*) For principal stresses, we have

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}{2} \qquad \text{...(Eq. 2.13)}$$

or $= \dfrac{50 - 35}{2} \pm \dfrac{\sqrt{(50+35)^2 + 4 \times 40^2}}{2}$ (σ_y *is compressive as such taken –ve*)

or $= 7.5 \pm 58.36$

or $\sigma_1 = +65.86\ \text{N/mm}^2$ (*tensile*)

and $\sigma_2 = -50.86\ \text{N/mm}^2$ (*Compressive*).

Graphically (*Using Mohr Circle method*)

To some convenient scale take OM_1 (Fig. 2.31) to the right of Y-axis to represent the tensile (+*ve*) stress of 50 N/mm^2 and OM_2 to the left of Y-axis to represent compressive (–*ve*) stress of 35 N/mm^2. Draw perpendiculars to X-axis at M_1 and M_2 and scale off M_1L_1 and M_2L_2 to represent shear stress of 40 N/mm^2.

Let the line L_1L_2 cut the X-axis in C with C as centre and CL_1 (=CL_2) as radius draw the Mohr's Circle cutting the X-axis in N and R. Now ON and OR represent the two principal stresses. Measure these and as per scale chosen work out the two stresses.

For inclination of planes measure $\angle M_1CL_1$ and $\angle M_1CL_2$. These are double the angles of the inclinations of the two principal planes to the direction of 50 N/mm^2 stress.

b) A shaft is subjected to a twisting moment which produces a shearing stress of 120kN/cm^2 at its surface in planes perpendicular to its axis. Taking C=8.6 x 106 N/cm^2, find changes in the angles of small square scratched on the surface of the shaft with two of its sides parallel to the axis of the shaft.

Solution:

Within limits of proportionality,

$$C = \frac{\tau}{\phi}$$

or

$$\phi = \frac{\tau}{C}$$

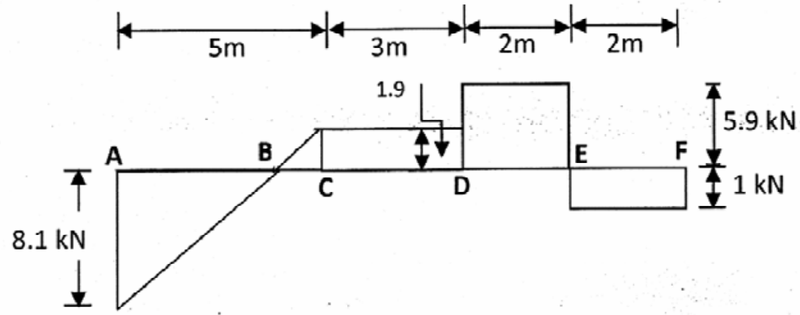
$$\therefore \quad \phi = \frac{12\times10^3}{8.6\times10^6} = 0.001395 \text{ radian}$$

But π radian = 180°

$$\therefore \quad 0.001395 \text{ radian} = \frac{180}{\pi}\times 0.001395 = 0.0799° = 4'48''$$

Therefore, change in angle ϕ = 4'48"

Q.3) a) The S.F. diagram for a beam ABCDEF, supported at A and at one more point is represented by the fig as shown. Draw the load and the B.M. diagram. Also locate the point of contra-flexure (if any)



Solution:

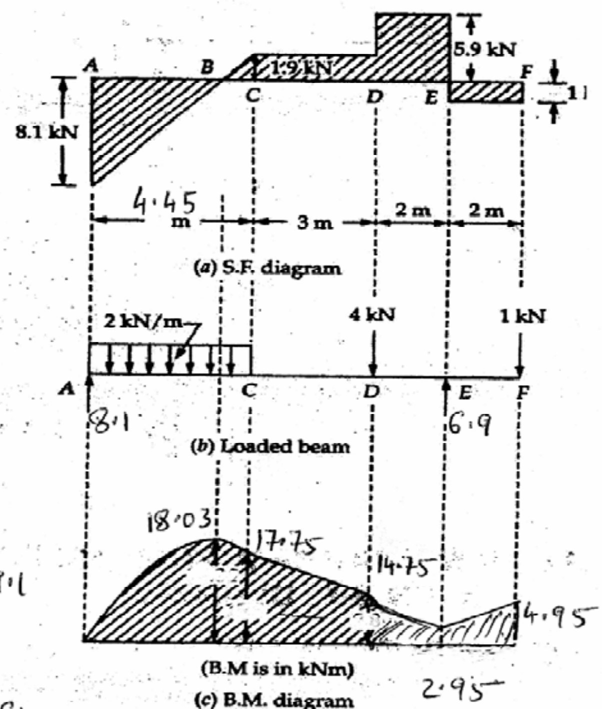
S.F. diagram: Straight line in the S.F. diagram between *F* and *E* in the S.F. diagram indicates that there is only one point load of 1.0 kN at *F* and no other load between *F* and *E*.

S.F. changes its nature at E and the magnitude by (1 + 5.9) = 6.9 kN. It remains unchanged between *E* and *D* which implies that a point load of magnitude 6.9 kN is acting at *E* in a direction opposite to that of 1 kN force acting at *F* and that there is no other force/ load between *E* and *D* (since the S.F. diagram is horizontal between *D* and *E*).

Sudden change in S.F. by (5.9 – 1.9 = 4.9 kN) indicates presence of a point load of 4.9 kN at *D* and horizontal line between *D* and *C* indicates absence of any load on *DC*.

From *C* to *A* the S.F. changes gradually from +1.0 kN to –8.1 kN. It indicates that part *CA* of the beam carries a U.D.L. of magnitude (1.0 + 8.1) = 9.1 10 kN, *i. e.*, at the rate of $\frac{9.1}{5}$ = 1.82 kN/m.

At *A* there is sudden application of 8.1 kN force.
Based on above the load diagram is as shown in fig.



Bending moment diagram:

M_E = –Area of S.F. diagram between *F* and *E* = –1 × 2 = –2 kNm

M_D = (Area of S.F. diagram between *E* and *D*) – (Area of S.F. diagram between *F* and *E*
= (5.9 × 2) – (1 × 2) = kNm 14.75

M_C = (Area of S.F. diagram between E and C) – (Area of S.F. diagram between F and E)

$= 14.75 \times 2 + 1.0 \times 3) - (1 \times 2) = 17.75$ kNm

From the two similar Δ's, we have:

$$\frac{1.9}{BC} = \frac{8.1}{5 - BC}$$

∴ $BC = 0.95$ m.

B.M. at B = (Area of S.F. diagram between E and B) – (Area of S.F. diagram between F and E)

or $$= \left(14.75 \times 2 + 1.0 \times 3 + \frac{1}{2} \times 1.0 \times 0.95\right) - (1 \times 2) = +18.75 \text{ kNm}$$

B.M. at A = (Area of S.F. diagram between E and B) – (Area of S.F. diagram between B and A)

or $$= \left(14.75 \times 2 + 1.0 \times 3 + \frac{1}{2} \times 1.0 \times 0.95\right) - \left[1 \times 2 - \frac{1}{2} \times 8.1 \times (5 - 0.95)\right] = 0$$

Load diagram and the B.M. diagram are shown in Fig. (b) and (c) respectively.

b) Draw the S.F. & B.M. diagrams for the beam ABCD (total span 12 m) supported at A, C & D. There is a internal hinge at B. Indicate the values at all salient points. Beam is loaded with a triangular loading between AB, length 6m (value of load is zero at B and varies to 6 kN/m at A). One point load of 6 kN is acting at B.UDL @ 8 kN/m is acting between CD with span as 4m.

Solution :

1. **Reactions** : Due to hinge at B, the B.M. M_B is zero.

Hence $M_B = 0 = 6\,R_A - \frac{1}{2} \times 6 \times 6 \left(\frac{2}{3} 6\right)$ which gives $R_A = 12$ kN (↑). Similarly, considering all the forces to the right of B, $M_B = 0 = R_D \,.\, 6 + R_C \,.\, 2 - 8 \times 4 \times 4$.

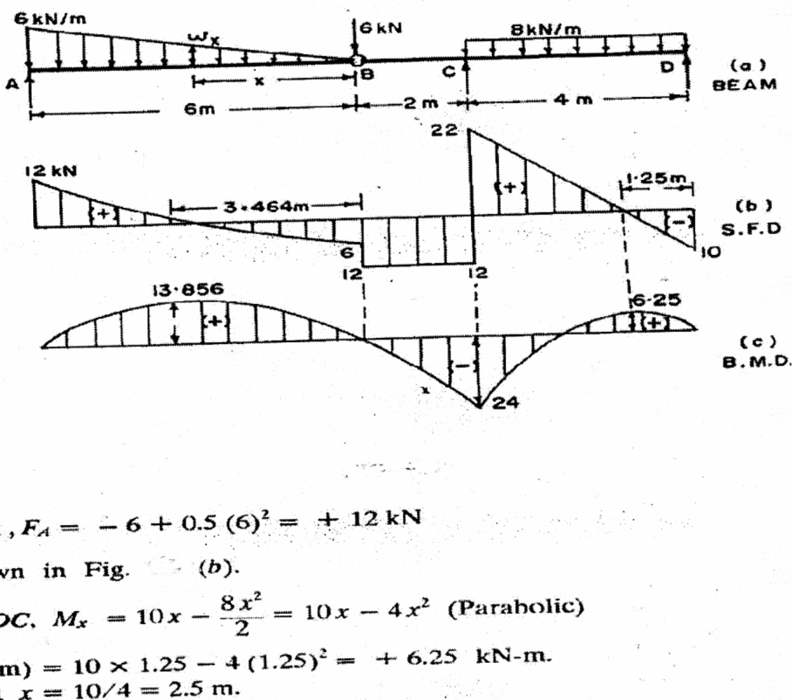
$$\therefore \quad 3\,R_D + R_C = 64 \qquad \text{...}(i)$$

Also, for the whole beam,

$$R_A + R_C + R_D = \left(\frac{1}{2} \times 6 \times 6\right) + 6 + (8 + 4)$$

or $$R_C + R_D = 56 - R_A = 56 - 12 = 44 \qquad \text{...}(ii)$$

From (*i*) and (*ii*) we get $R_D = 10$ kN and $R_C = 34$ kN.



2. **S.F. Diagram:** ***For DC.*** Measuring x from D, F_x $= -10 + 8x$, which gives $F_D = -10$ kN and F_C (*right*) $= -10 + 8 \times 4 = +22$ kN. S.F. is zero at $x = 10/8 = 1.25$ m.

For CB : $F_x = -10 + 32 - 34 = -12$ kN, which is constant for CB.

For BA : Measuring x from B. $w_x = \frac{6}{6} x = x$ kN/m

$$\therefore F_x = -10 + 32 - 34 + 6 + \frac{1}{2} x \,.\, x$$

$= -6 + 0.5x^2$, which is a parabolic variation.

S.F. is zero at $x = \left(\frac{6}{0.5}\right)^{1/2} = 3.464$ m. At $= 6$ m, $F_A = -6 + 0.5\,(6)^2 = +12$ kN

The S.F. diagram is shown in Fig. (*b*).

3. **B.M. Diagram** : ***For DC***, $M_x = 10x - \frac{8x^2}{2} = 10x - 4x^2$ (Parabolic)

$$M_{max} \text{ (at } x = 1.25 \text{ m)} = 10 \times 1.25 - 4\,(1.25)^2 = +6.25 \text{ kN-m.}$$

B.M. is zero at $x = 10/4 = 2.5$ m.

Also, at $x = 4$, $M_C = 10 \times 4 - 4\,(4)^2 = -24$ kN-m

For CB : Measuring x from C, $M_x = 10\,(4 + x) - 32\,(2 + x) + 34\,x = 12\,x - 24$ which is a linear variation. At $x = 2$ m, $M_B = 12 \times 2 - 24 = 0$, as expected.

For BA : Measuring x from B, $M_x = 10\,(6 + x) - 32\,(4 + x) + 32\,(2 + x) - 6x - \frac{1}{2} x \,.\, x \,.\, \frac{x}{3} = 6x - \frac{x^3}{6}$, which is a cubic curve. Maximum occurs at $x = 3.464$ m, where S.F. is zero. Hence $M_{max} = 6 \times 3.464 - \frac{(3.464)^3}{6} = 13.856$ kN-m. At $x = 6$ m, $M_A = 6 \times 6 - \frac{(6)^3}{6} = 0$ as expected. The B.M.D. is shown in Fig. (*c*).

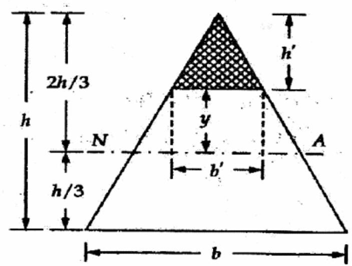
Q.4) a) A beam has triangular cross-section with base 'b' and height 'h' and is used with the base horizontal. Calculate the intensity of maximum shear stress and plot the variation of shear stress intensity along the section.

Solution:

The centroid of the triangle is at $\frac{h}{3}$ from the base (

Let the shear stress at a height y above the N.A. be τ. If b' and h' be the base width and height respectively of the shaded Δ then

$$\frac{b'}{h'} = \frac{b}{h}$$



$$\therefore \quad b' = \left(\frac{b}{h} \times h'\right)$$

But $$h' = \left(\frac{2h}{3} - y\right)$$

$$\therefore \quad b' = \frac{b}{h}\left(\frac{2h}{3} - y\right)$$

Height of the centroid of the shaded Δ from the N.A. is

$$\left(y + \frac{h'}{3}\right) = y + \frac{1}{3}\left(\frac{2h}{3} - y\right) = \frac{2}{3}\left(y + \frac{h}{3}\right)$$

Moment of the shaded area about N.A.

$$= \frac{b'h'}{2} \times \frac{2}{3}\left(y + \frac{h}{3}\right) = \left[\frac{1}{2} \times \frac{b}{h}\left(\frac{2h}{3} - y\right) \times \left(\frac{2h}{3} - y\right)\right] \times \left[\frac{2}{3}\left(y + \frac{h}{3}\right)\right]$$

or $$= \frac{b}{3h}\left(\frac{2h}{3} - y\right)^2 \times \left(y + \frac{h}{3}\right)$$

$$\therefore \quad \tau = \frac{F}{Ib'} \times \frac{b}{3h}\left(\frac{2h}{3} - y\right)^2 \times \left(y + \frac{h}{3}\right)$$

or $$= \frac{F \times h}{I \times b\left(\frac{2h}{3} - y\right)} \times \frac{b}{3h}\left(\frac{2h}{3} - y\right)^2 \times \left(y + \frac{h}{3}\right)$$

or $$= \frac{F}{3I}\left(\frac{2h}{3} - y\right)\left(y + \frac{h}{3}\right) = \frac{F}{3I}\left(\frac{hy}{3} - y^2 + \frac{2h^2}{9}\right) \quad \text{...(i)}$$

For τ to be maximum, put

$$\frac{d\tau}{dy} = 0$$

Therefore, for τ_{max} we have

$$\frac{h}{3} - 2y = 0$$

or $$y = \frac{h}{6}$$

Height from the base of the plane on which shear stress is maximum is

$$\left(\frac{h}{3} + \frac{h}{6}\right) = \frac{h}{2}$$

Therefore, from Eq. (i)

$$\tau_{max} = \frac{F}{3I} \times \left[\frac{h}{3} \times \frac{h}{6} - \left(\frac{h}{6}\right)^2 + \frac{2h^2}{9}\right] = \frac{Fh^2}{12I}$$

or $$= \frac{Fh^2}{12 \times \frac{bh^3}{36}} = \frac{3F}{bh}$$

For $\tau_{N.A.}$ we have $y = 0$

Therefore, from Eq. (i),

$$\tau_{N.A.} = \frac{F}{3I}\left(\frac{2h^2}{9}\right) = \frac{2Fh^2}{27 \times \left(\frac{bh^3}{36}\right)} = \frac{8F}{3bh}$$

Q.4. b) State the Mohr's theorems. A horizontal beam rests on two supports at the same level and carries a UDL. If the supports are symmetrically placed, find their positions when the greatest downward deflection has its least value. Use Moment Area method.

Solution :

Mohr's Theorems:

Theorem I. The angle in radians between the tangents to the elastic curve at two points on a straight member under bending is equal to the area of the $\frac{M}{EI}$ diagram between those two points.

Theorem II. The deflection of a point on a straight member under bending in the direction perpendicular to the original straight axis of the member, *measured from the tangent at another point on the member,* is equal to the moment of the *M/EI* diagram between those two points, about the point where this *deflection occurs.*

Numerical:

Let distance between the two supports be $2l$ and overhangs on either side be a each . If the rate of the loading on the beam be w/unit length then each of the two the support reactions are $R_C = R_D = (wl + a)$.

Ends A and B as also the midpoint of the span CD are likely to have downward deflections. For the maximum downward deflection anywhere to be the least, it is essential that the deflections at

A or B and at K, the midpoint of CD, should be equal. Because of symmetrical loading, the tangent to the elastic curve at K shall be horizontal and since deflections at K and A are equal, the deflection of A with reference to the tangent to the elastic curve at K shall be zero. Therefore, moment of the B.M. diagram, between A and K, about A should be zero. Figure shows the bending moment due to U.D.L. and support reaction R_C separately only for half the span (for convenience).

$$\text{Area } bde \times \left(ab - \frac{l}{3}\right) - \text{Area } abc \times \left(ab - \frac{l+a}{4}\right) = 0$$

$$\therefore \quad \left(\frac{wl(l+a)}{2} \times l\right) \times \left(l + a - \frac{l}{3}\right) - \frac{1}{3}\frac{w(l+a)^2}{2} \times (l+a) \times \left(l + a - \frac{l+a}{4}\right) = 0$$

$$\therefore \quad 3a^3 + 9a^2l - 3al^2 - 5l^3 = 0$$

$$\text{or} \quad 3\left(\frac{a}{l}\right)^3 + 9\left(\frac{a}{l}\right)^2 - 3\left(\frac{a}{l}\right) - 5 = 0$$

Since $\left(\frac{a}{2l}\right)$, the ratio of length of overhang to that of supported length is required, let

$$\frac{a}{2l} = x$$

$$\therefore \quad \frac{a}{l} = 2x$$

$$\therefore \quad 24x^2 + 36x^2 - 6x - 5 = 0$$

By trial and error, $x = 0.403$

$$\frac{a}{2l} = \frac{AC}{CD} = 0.403$$

Q.5) a) During tests on a sample of steel bar 25 mm in diameter, it is found that the pull of 50 kN produces an extension of 0.095 mm on a length of 200 mm and a torque 200-N-m produces an angular twist of 0.9 degrees on a length of 250 mm. Find the poisson's ratio of the steel.

Solution.

$$d = 25 \text{ mm}$$

$$\text{Pull} = 50 \text{ kN}$$

$$\delta_1 = 0.095 \text{ mm}$$

$$\text{Gauge length} = 200 \text{ mm.}$$

Using the expression for extension of a bar under direct force, we have

$$0.095 = \frac{50 \times 10^3 \times 200}{\frac{\pi}{4} \times 25^2 \times E}$$

or

$$E = 214440.34 \text{ N/mm}^2$$

Now, from torsion experiment

$$T = 200 \text{ N-m} = 200 \times 10^3 \text{ N-mm}$$

b) A straight bar of alloy, with pin-jointed ends, is 1.5 m long and 15mm x 10 mm in section. It is mounted in a strut-testing machine and loaded axially till it buckles. Using Euler's formula, calculate the maximum central deflection before the material attains its yield point at 300 MN/m^2. Take E= 80 GN/m^2.

Solution :

For the bar section $I = \frac{15 \times 10^3}{12} = 1250 \text{ mm}^4 = 1.25 \times 10^{-9} \text{ m}^4$ (taking least value of I)

Section modulus $Z = \frac{I}{y} = \frac{1.25 \times 10^{-9}}{5 \times 10^{-3}} = 2.5 \times 10^{-7} \text{ m}^3$

For pin-jointed ends, Euler's load is

$$P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times (80 \times 10^9) \times (1.25 \times 10^{-9})}{1.5^2} = 438.65 \text{ N}$$

Direct compressive stress due to load P is

$$\sigma_d = \frac{P}{A} = \frac{438.65}{15 \times 10} = 2.92 \text{ N/mm}^2 = 2.92 \times 10^6 \text{ N/mm}^2$$

If the central deflection is y then

$$M_{max} = Py = 438.65\, y$$

Maximum bending stress is

$$\sigma_b = \frac{M}{Z} = \frac{438.65y}{2.5 \times 10^{-7}} = 175.46 \times 10^7\, y$$

But $\sigma_d + \sigma_b = 300 \times 10^6$

or $2.92 \times 10^6 + 175.46 \times 10^7 y = 300 \times 10^6$

or $y = 0.169 \text{ m}$

Q.6) a) A steel cylindrical shell of internal diameter 25 cm and has to withstand internal pressure of 47.5 N/mm^2. Calculate the metal thickness required if the maximum permissible tensile stress is 126 N/mm^2.

Solution:

Now as per Lame's equations,

$$\sigma_i = \frac{b}{r_i^2} - a$$

or $$4750 = \frac{b}{12.5^2} - a \quad ...(i)$$

and $$\sigma_{c\,max} = \frac{b}{r_i^2} + a$$

or $$12600 = \frac{b}{12.5^2} + a \quad ...(ii)$$

By elimination from the two equations (i) and (ii), we have,

$$a = 3925 \text{ N/cm}^2 \quad \text{and} \quad b = 1355.47 \times 10^3 \text{ N}$$

Apply Lame's equation to outer surface of cylinder where $r = r_o$ and $\sigma_o = 0$

$$\therefore \quad 0 = \frac{b}{r_o^2} - a$$

or $$r_o^2 = \frac{b}{a} = \frac{1355.47 \times 10^3}{3925}$$

$$\therefore \quad r_o = 18.6 \text{ cm}$$

Metal thickness $= (r_o - r_i) = 18.6 - 12.5 = 6.1$ cm

b) i) Discuss different types of Riveted & Welded connections.

Types of welded Connections:

Welded joints are primarily of two kinds

a) **Lap or fillet joint**: obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints.

b) **Butt joints**: It is formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g., Square butt joint, Single V-butt joint, double V-butt joint, Single U-butt joint, double U-butt joint, Single J-butt joint, double J-butt joint, Single bevel-butt joint, double bevel butt joint,Double – V butt joint Single – V butt joint Square butt joint . There are other types of welded joints, for example, Corner joint, Edge or seal joint and T-joint

Types of Rivetted Connections:

A riveted joint may be classified according to (a) arrangement of rivets and plates (b) mode of load transmission, and (c) nature and location of load with respect of rivet group.

(a) Arrangement of Rivets and Plates.

According to the arrangement of rivets and plates, riveted joints may be of the following types:

(1) *Lap Joint*:

(i) Single riveted

(ii) double riveted.

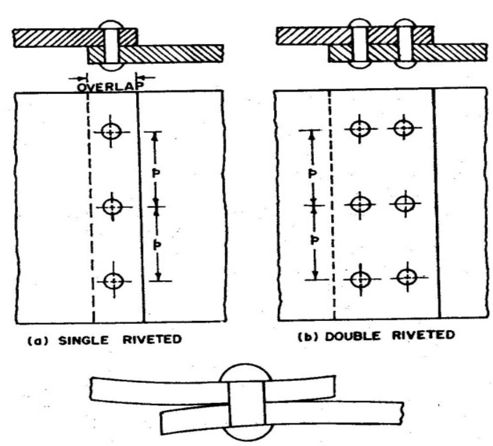


FIG. 26.5. LAP JOINTS

(2) *Butt joint:*

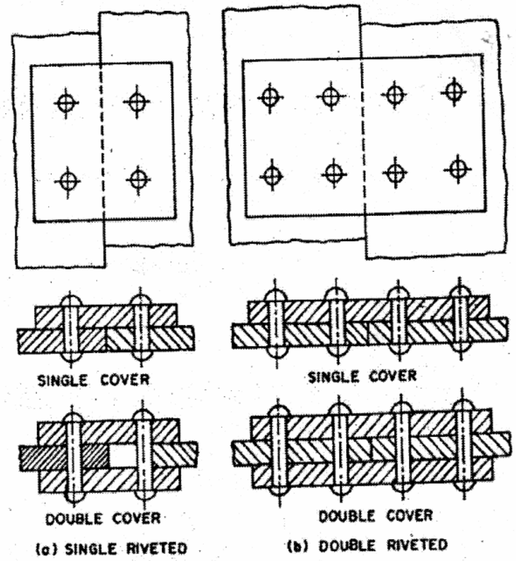
(*i*) Single riveted butt joint with single cover plate.

(*ii*) Single riveted butt joint with double cover plate

(*iii*) Doubled riveted butt joint with double cover plate.

Fig. (*a*) shows single riveted lap joint, while (*b*) shows double riveted lap joint. The two lines of pull in the joined plates are not in alignment, resulting in bending stresses tending to distort the joint, as shown in Fig. 26.5 (*c*).

Fig. (*a*) and (*b*) show single riveted butt joint and double riveted butt joint respectively, in which the edges of the plates come flush and the *cover plates* are used to join them. In a single cover butt joint, bending stresses may develop, tending to distort the joint, as shown in Fig. This possibility is completely eliminated by using a double cover butt joint.

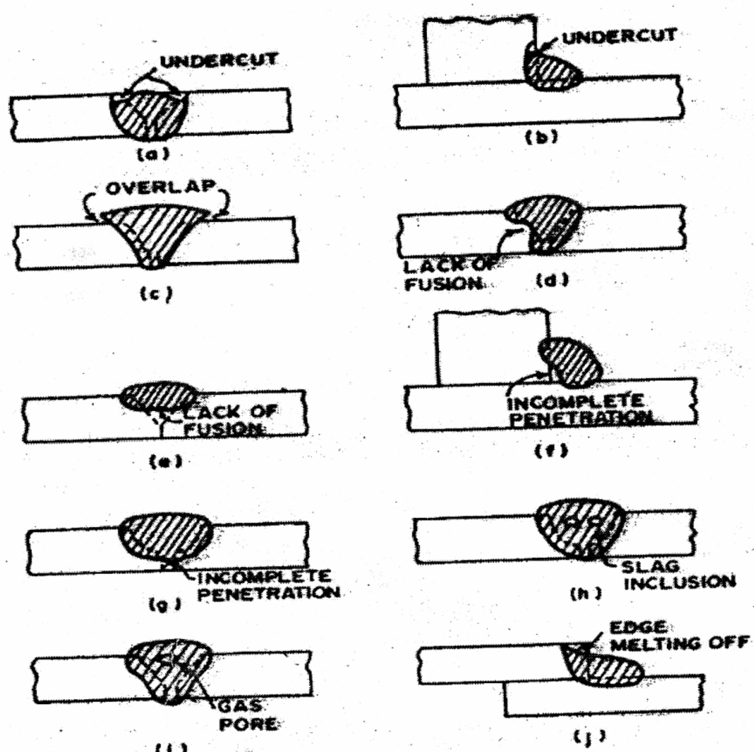


ii)Describe important weld defects.

Welding is highly specialised technique of jointing, and it should be done carefully so that no *defects* or *imperfections* are left. The most important defects arising from the welding technique are as follows:

1. Undercutting
2. Overlap
3. Incomplete penetration
4. Lack of fusion
5. Slag inclusion
6. Porosity and gas inclusion
7. Edge melting

These defects have been shown diagrammatically in Fig. . *Under cutting* takes place due to excessive current and excessive length of arc, resulting in the formation of a groove in the base metal. When the weld metal overflows the groove, but does not fuse with base metal, and *overlap* is formed . *Incomplete penetration* takes place when the weld metal does not penetrate up to the root of the joint because of faulty groove preparation or because of faulty technique used during welding. *Lack of fusion* takes place when the parent metal is coated with some foreign matter and when the groove is not clean. Due to this, there will be lack of union between two runs of weld metal. *Slag inclusion* takes place because of formation of oxides due to chemical reaction among the base metal, air and electrode coating, during welding. Some times, a group of *gas pores* may get entrapped in the weld, as shown in . Such a defect of *gas inclusion* is also called *porosity*. *Edge melting off* occurs in fillet welds because of careless welding.



IMPORTANT WELD DEFECTS.